WHEN, WHERE, WHY & HOW?

EXTENDED DEGREES:

MATHS IN PRACTICE &

PRACTICE AS MATHEMATIST

DR RUAN MOOLMAN

UCT

DEPT MATHS & APPLIED MATHS

TWO FOCI OF TALK

Applications of Maths
↔
Motivation
↔
Learning Performance

Mathematist/Scientist/
Engineer
Practicing Maths
Mathematics Knower
↔
Metacognitive Skills
(Critical Thinking Skills)
(Problem Solving Skills)

IMPROVE
method

Metacognitive
Questions

Assist with math
problem solving

BACKGROUND

MAM1000W

MAM1005H

MAM1006H

Extended Degree Maths

Students:

Whole Range of BSc, Actuaries, Financial Maths, Analytical Science, Computer Science with Maths 2nd year/3rd year

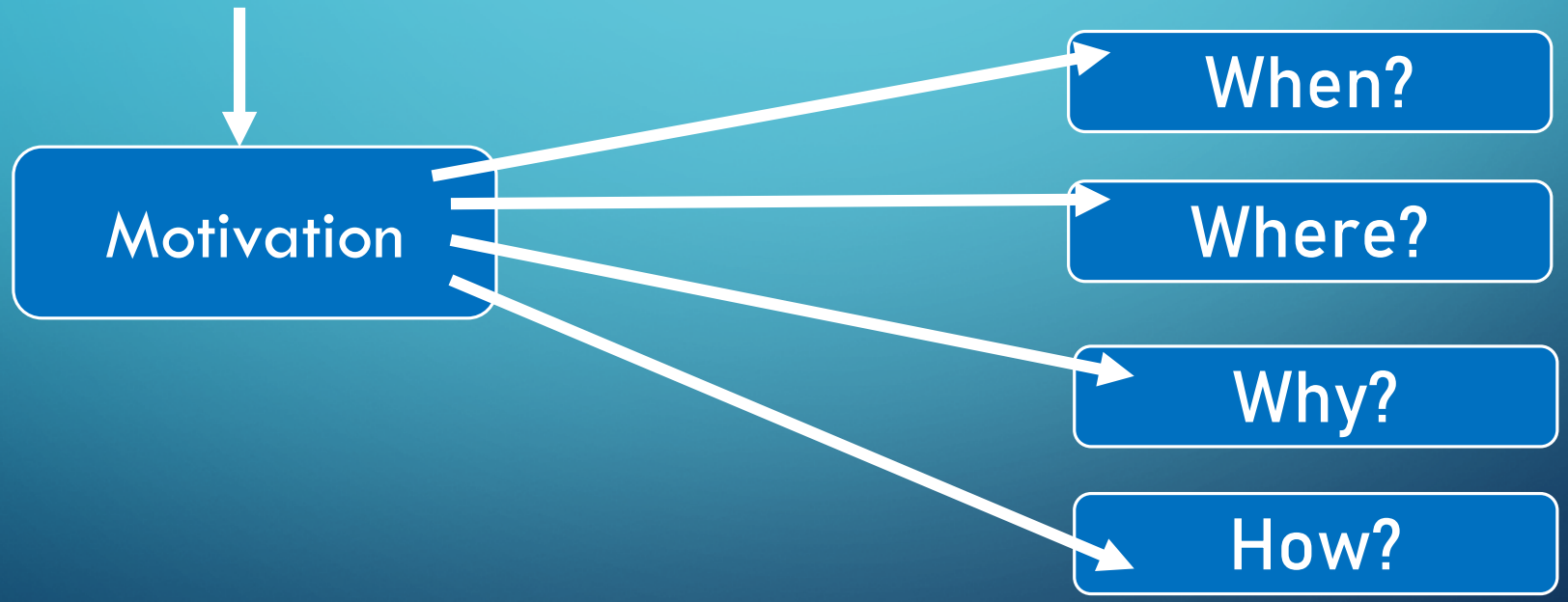
Half Year Course

Whole Year Course

Need for 2nd/3rd Year Maths/Applied Maths

BACKGROUND

- Small group who major in maths and/or applied maths
- Majority students: service course(s) → just do the maths
- Must do maths ↔ 'Chore' ↔ 'Punishment'



Motivation (Voight, 2002)

- **Indicate the usefulness of a topic** (George Lucas Educational Foundation, Edutopia)
- 'In many ways, motivation is a critical undercurrent in the learning environment and can be a crucial determinant of student achievement.' John Voight
- Motivating the material by providing **context** is a great way to motivate students to investigate it for themselves.

Motivation (Voight) Contd.

- "It is our job as teachers of mathematics to introduce students to this exciting field, and to motivate students to want to study mathematics and to major in it. Applications are a device for achieving this end."
- How is this achieved: adapt an appropriate level of challenge (tasks, exercises, examples)
- How determine level?
- Instructor to know class:

  their majors,

  their expectations,

  their backgrounds.

  INTERACTION (CONSTANT FEEDBACK, ONGOING PROCESS)
- Similar results/discussion in Nguyen & Goodin (2016)

  As educator:

  - Choosing a task is important,

  - Integration of these tasks also important.

Study: University Students' Views of the Role of Maths in their Future

(Wood et al., 2012, Australia, South Africa, UK, Tanzania)

- Students asked:

  What part do you think mathematics will play in your future studies? … in your future career?

- Results and discussions:

For students from professional disciplines, **spelling out how mathematics fits** into their degree courses **may assist with their learning**.

Study: University Students' Views of the Role of Mathematics in their Future (Wood et al., 2012, Australia, South Africa, UK, Tanzania)

- Results and discussions:

Macbean (2004) recommends that: the more **students believe** the mathematics is **integrated** and **integral** to their degree course the more **motivated** they are likely to be, and the more **meaning oriented their approaches to studying it will become**.

Consequently, the more a **department promotes** mathematics as important for the students' degree course and **adapts the teaching of it appropriately**, the more **meaning oriented** the students' **approach to studying** it will be, and the more likely it is for them to **develop cohesive conceptions** of mathematics.

Study: University Students' Views of the Role of Mathematics in their Future (Wood et al., 2012, Australia, South Africa, UK, Tanzania)

- Macbean notes that this is particularly important for students studying mathematics as a **service subject** since **context** and perceived relevance will **greatly affect** their **motivation** and, hence, their **learning**.
- E.g. of evidence from Wood study supports this *"I'd like to be able to relate it to something that's a real life problem, that's one thing that I find with maths, if I can't relate it to a real life situation, then I don't see a point."* (Eddie, third year)

Initiative in SA (Hockman, 2006)

AIMS: African Institute for Mathematical Sciences

Programme of recruiting and training African *graduate* students in a broad range of mathematical sciences and mathematical sciences research skills, thereby preparing them for careers in research, education, industry or government.

Students response to AIMS:

- They **felt** that learning the **mathematics necessary** to solve the real-world problems that were posed, at the time of need, **produced greater insight and understanding into the mathematics than they had had previously.**
- The students understood that mathematics is a '**powerful tool' that can be used to solve real-world problems.** They felt that AIMS had **changed their perspective** of mathematics **from an abstract** subject composed of theorems and algorithms to a **worthwhile enterprise** that can address, **solve** and throw insight on important **physical problems** and hence contribute **meaningfully to society**.
- They felt that the experience at AIMS would **affect** the way that **they interacted with students and with mathematics in the future**. Furthermore they felt that the experience had gone some way in **training** them **to conduct independent research and to behave like professional mathematicians.** They were inspired to study more and to **contribute back to society.**

Extended Maths UCT

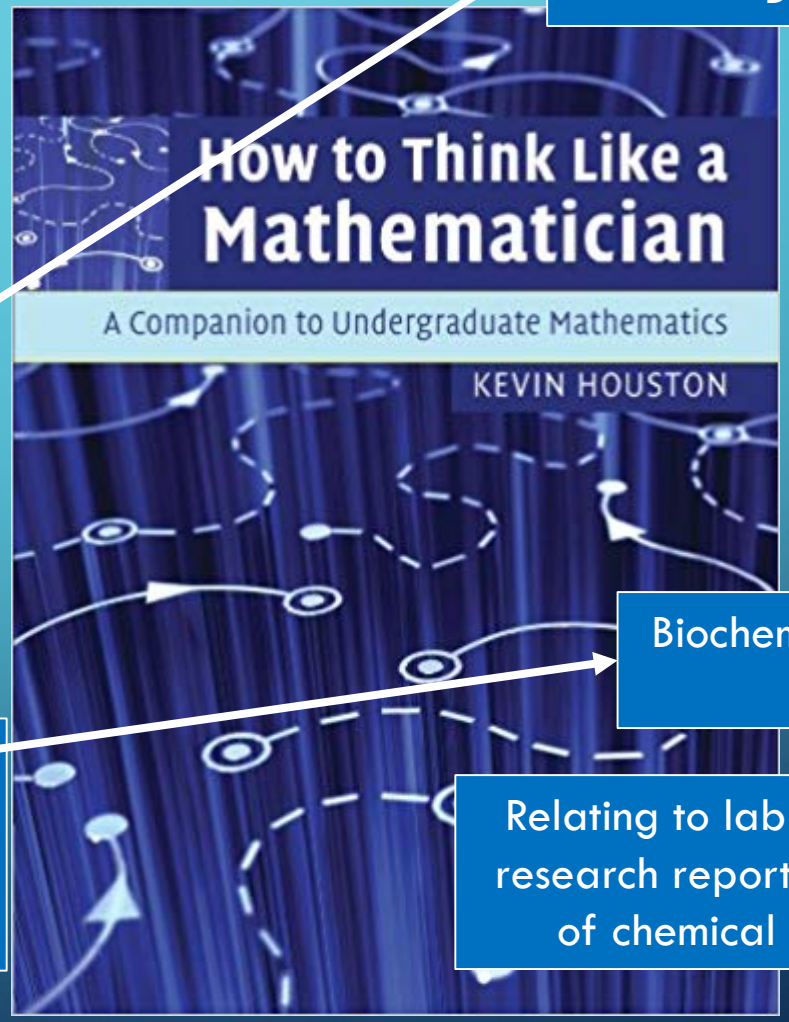
Logic/Truth Tables/Implications

Basis argumentation/ justification/proof: 2nd & 3rd year Maths & Applied Maths

$$A \Rightarrow B$$
$$B \Rightarrow A$$
$$\neg B \Rightarrow \neg A$$
$$\neg(A \Rightarrow B) \equiv A \wedge \neg B$$

Actuarial Sciences

Linear Algebra
BVP and DE



Biochemistry/Chemistry student asking why?

One year maths:
Proof by Induction
and
Proof by Contradiction

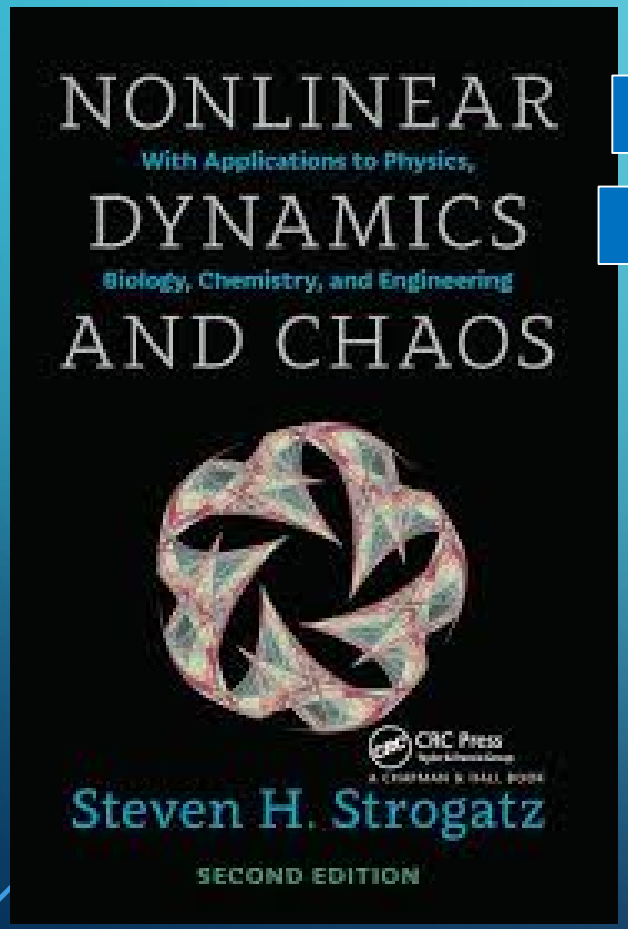
Relating to lab experiments / research reports / future work of chemical engineering

Differential Equations

What do I need to know?

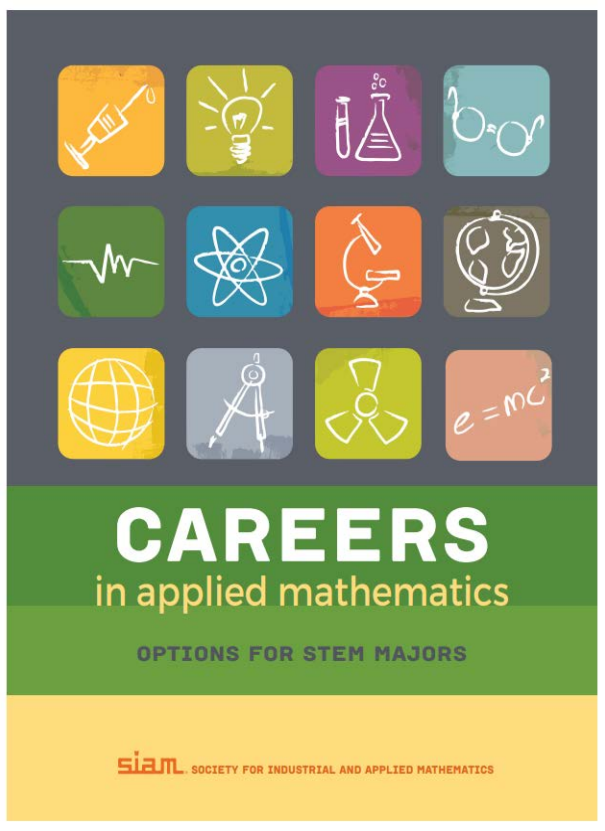
1. Derivatives

2. Integration



Applied Maths

AIMS



IN THIS GUIDE, YOU WILL FIND ANSWERS TO QUESTIONS ABOUT CAREERS IN APPLIED MATHEMATICS AND COMPUTATIONAL SCIENCE, AND PROFILES OF PROFESSIONALS WORKING IN A VARIETY OF ENVIRONMENTS FOR WHICH A STRONG BACKGROUND IN MATHEMATICS IS NECESSARY FOR SUCCESS.

HERE ARE SOME EXAMPLES OF ORGANIZATIONS THAT HIRE MATHEMATICIANS AND COMPUTATIONAL SCIENTISTS:

- ACADEMIC INSTITUTIONS AND RESEARCH INSTITUTES
- AEROSPACE AND TRANSPORTATION EQUIPMENT
- OR SERVICE PROVIDERS
- FORECASTING ORGANIZATIONS
- ARMACEUTICAL MANUFACTURERS
- IONS SERVICES PROVIDERS
- NFORMATION AND SOFTWARE FIRMS;
- D OR START-UPS
- DUCTS COMPANIES
- S FIRMS
- D COMPUTER MANUFACTURERS
- ESEARCH ORGANIZATIONS
- VICE AND INVESTMENT MANAGEMENT FIRMS
- ABS, RESEARCH OFFICES AND AGENCIES
- PANIES
- E COMPANIES
- PRODUCERS OF PETROLEUM AND PETROLEUM PRODUCTS

Show examples – cf. other resources/textbooks

Do examples – step-by-step

Discuss examples briefly in terms of applications, but also showing what maths used and done

Examples done in class basis for assignments

BSc

Physics/Financial Maths/Analytics/Com Sci

Actuaries/Financial Maths/Analytics/Com Sci

Chem/Biochem/Genetics/Microbio /Health Sciences

Actuaries/Financial Maths/Analytics/Com Sci

BSc combined Psych/Sociology/Industrial Psych

Geology/Enviro & Geo Impact/Botany/Zoology

Chem/Biochem/Genetics/Microbio /Health Sciences/ Anatomy/Physiology

Geology/Enviro & Geo Impact/Chem/Physics/Com Sci

Third Year and Honours Maths / Real Analysis

BVP & DE

Operator Theory

Physics:

Oscillations, harmonic behaviour, electromagnetics (Maxwell laws), fluid dynamics, quantum mechanics

Linear Algebra

System of linear equations →

System of diff eqtns →

Electrical Circuits (Kirchoff's Laws)

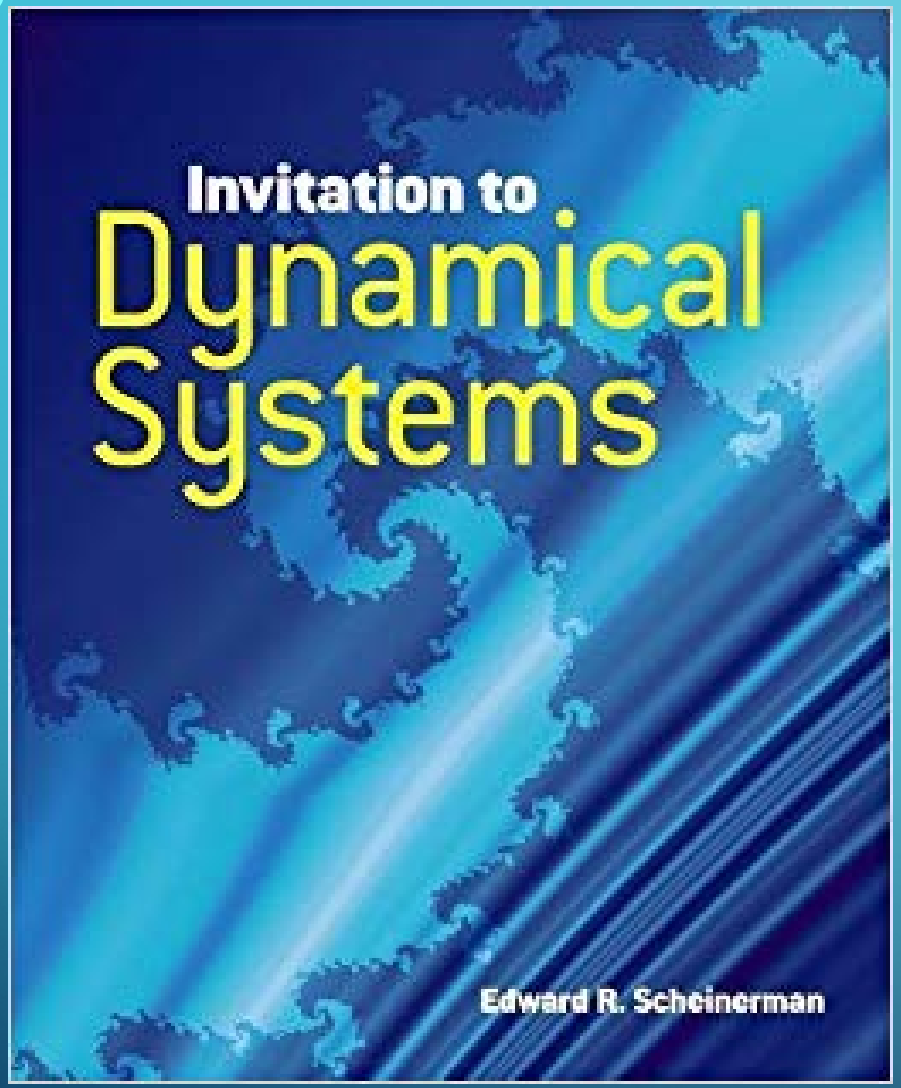
Eigenvectors, Eigenvalues, Eigenspaces

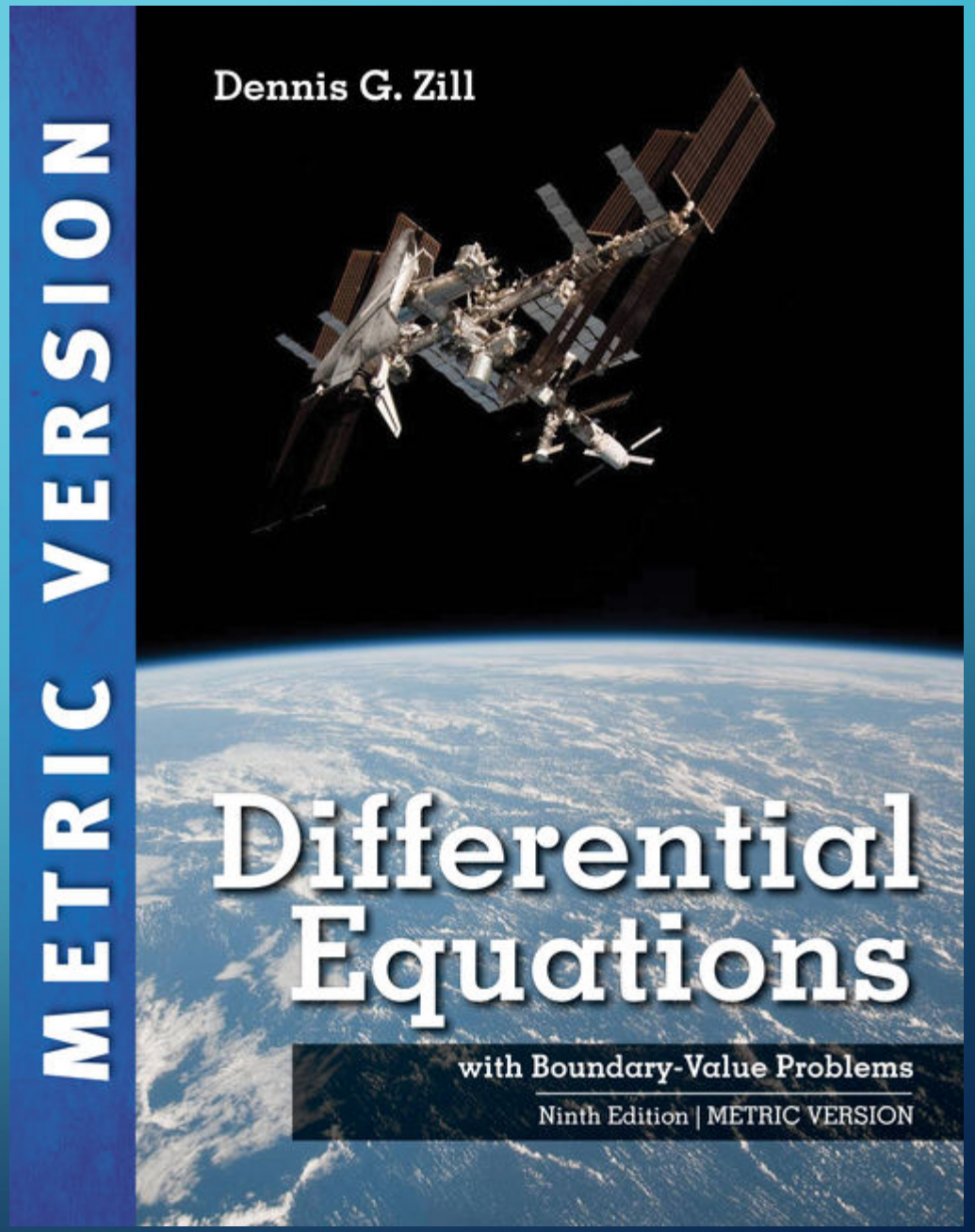
Hermitian Operators and Inner Product Spaces

Quantum Mechanics

Our world / Technology

Chemistry 3rd Year
Quantum Chemistry

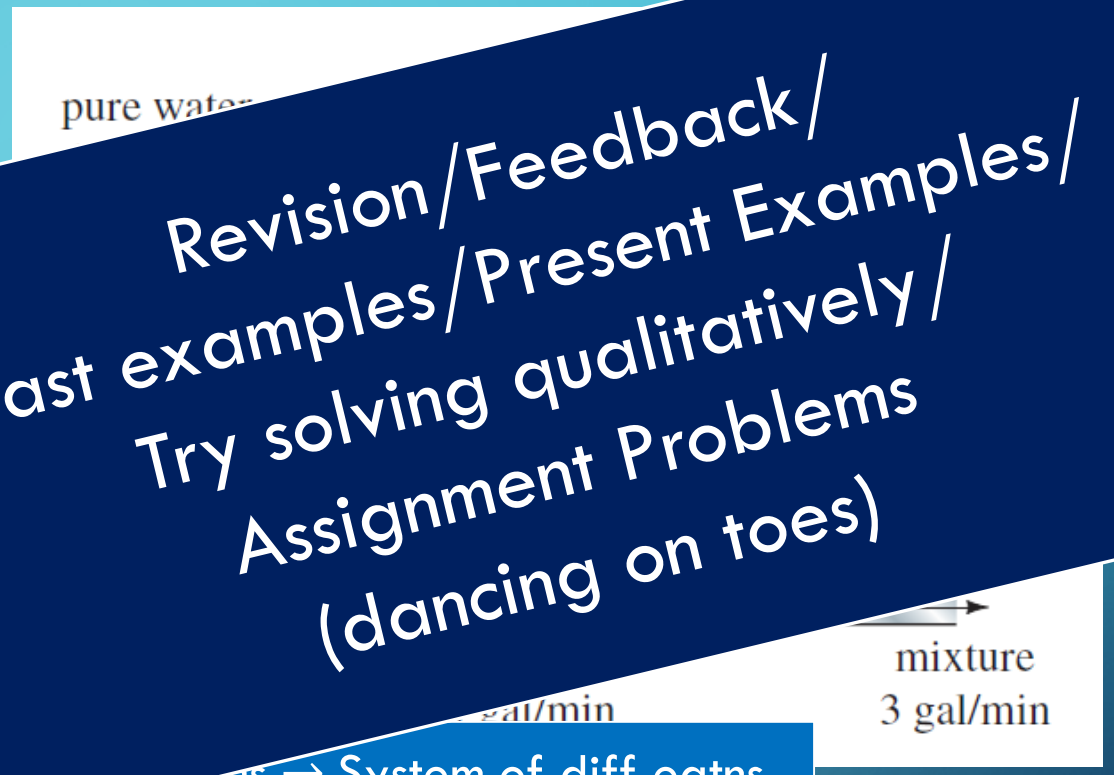

Invitation to
Dynamical
Systems
Edward R. Scheinerman


METRIC VERSION
Dennis G. Zill
Differential
Equations
with Boundary-Value Problems
Ninth Edition | METRIC VERSION

Chemistry 3rd Year Quantum Chemistry

Mixture Problems, Chemical Reactions

Revision/Feedback/
Past examples/Present Examples/
Try solving qualitatively/
Assignment Problems
(dancing on toes)



$$\frac{dx_1}{dt} = -\frac{2}{25}x_1 + \frac{1}{50}x_2$$

$$\frac{dx_2}{dt} = \frac{2}{25}x_1 - \frac{2}{25}x_2.$$

System of lin ... ons → System of diff eqtns

Separable DE

Thermodynamics / Analytical Chemistry

Solutions of DE: Logs / exponentials / their graphs

Qualitatively / Figures

Assignments

- Holiday work / open book/ group work
- Problems more advanced and time not in class/tests/exams

. In Exercise 9.1.15 we formulated a model for learning in the form of the differentia

where $P(t)$ measures
skill after a training ti
formance, and k is a p
equation to find an ex
expression?

A glucose solution is administered intravenously into the bloodstream at a constant rate r. As the glucose is added, it is converted into other s
bloodstream at a rate th
at that time. Thus a moc
the glucose solution in t

where k is a positive co
(a) Suppose that the co
mine the concentrat
ential equation.
(b) Assuming that C_0 <
your answer.

A model for tumor growth is given by the Gompertz equation

$$\frac{dV}{dt} = a(\ln b - \ln V)V$$

where a and b are positive constants and V is the volume of the tumor measured in mm^3.

(a) Find a family of solutions for tumor volume as a function of time.
(b) Find the solution that has an initial tumor volume of $V(0) = 1\ mm^3$.

Prep for Assignments (Class Work)

Population Growth

$$\frac{dP}{dt} = kP$$

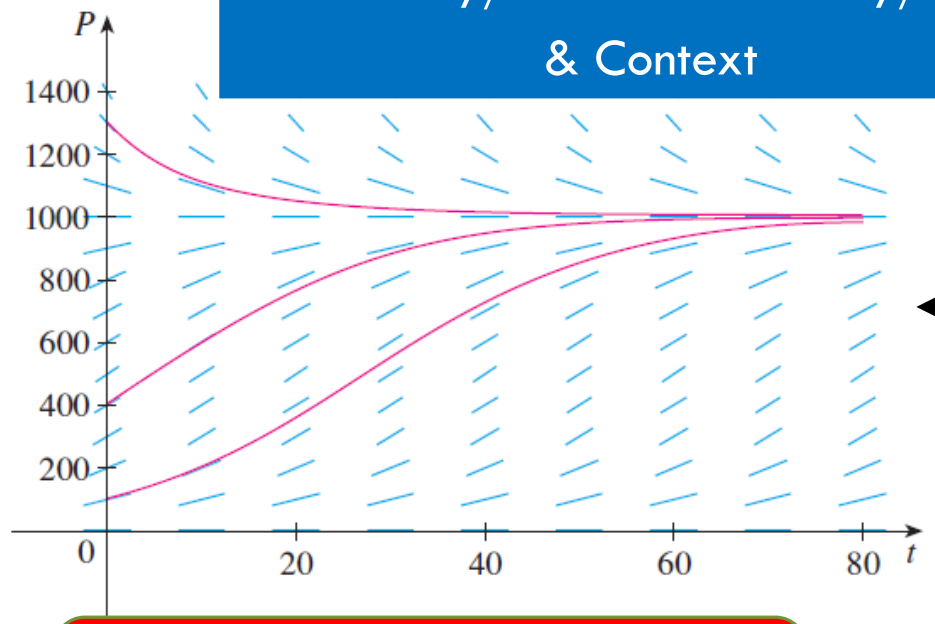
$$\frac{dP}{dt} = kP\left(1 - \frac{P}{M}\right)$$

Logistic Diff Eqtn

Separable DE

Transformations/substns to form linear 1st order DE

Vocabulary/Academic Literacy/Words & Context



$$P(t) = \frac{M}{1 + Ae^{-kt}} \qquad \text{where } A = \frac{M - P_0}{P_0}$$

Desmos/Geogebra/Bluffington University

Qualitative/Graphical

Analytical Solution

Assignments growing, expanding…

Two of the other models are modifications of the logistic model. The differential equation

$$\frac{dP}{dt} = kP\left(1 - \frac{P}{M}\right) - c$$

has been used to model populations that are subject to harvesting of one sort or another. (Think of a population of fish being caught at a constant rate.) This equation is explored in Exercises 19 and 20.

For some species there is a minimum population level m below which the species tends to become extinct. (Adults may not be able to find suitable mates.) Such populations have been modeled by the differential equation

$$\frac{dP}{dt} = kP\left(1 - \frac{P}{M}\right)\left(1 - \frac{m}{P}\right)$$

where the extra factor, $1 - m/P$, takes into account the consequences of a sparse population (see Exercise 21).

UCT Dept MAM

Marine Research Unit

By the procedure given in the subsection "Second-Order ODEs" we then find the general solution for $x(t)$ to be

Typical Solution & its Properties

$$x(t) = c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t}, \tag{42a}$$

where

Auxiliary eqtn

$$\lambda_{1,2} = \frac{\beta \pm \sqrt{\beta^2 - 4\gamma}}{2}. \tag{42b}$$

High School

The quantity $\delta = \beta^2 - 4\gamma$ is called the *discriminant*. (When δ is negative, eigenvalues are complex.) $y(t)$ can be found by solving (40a); see problem 21b. (Again, in the cases of comple the form of the general solution must be amended as befo

Linear Algebra, Complex Numbers/Analysis

Matrices as Linear Operators

Linear Operators & Eigenvalues nice properties

eneralisation

$$\mathbf{A}\mathbf{v} = \lambda\mathbf{v}.$$

Note: $\mathbf{v}$ cannot be "cancelled" since $\mathbf{A}\mathbf{v}$ stands for matrix multiplication; however, we can rewrite (45a) as

Future work: BVP and DE and Physics

$$\mathbf{A}\mathbf{v} - \lambda\mathbf{I}\mathbf{v} = 0,$$

where $\mathbf{I}$ is the identity matrix ($\mathbf{I}\mathbf{v} = \mathbf{v}\mathbf{I} = \mathbf{v}$). Now $\lambda\mathbf{I}$ is also a matrix, namely

a)

$$\begin{pmatrix} \lambda & & & & 0 \\ & \lambda & & & \\ & & \lambda & & \\ & & & \ddots & \\ 0 & & & & \lambda \end{pmatrix}$$

(43b)

(43c)

Thus equation (45b) can be written as

$$(\mathbf{A} - \lambda\mathbf{I})\mathbf{v} = 0. \qquad (45c)$$

Readers familiar with linear algebra will recogni

Determinants – current course & future courses

satisfied with eigenvalues λ and eigenvectors $\mathbf{v}$ of

trivial solution $\mathbf{v} = 0$, we must have

$$\det(\mathbf{A} - \lambda\mathbf{I}) = 0. \qquad (46)$$

Responses from students

Letters/E-mails/Reference Letters/Student Evaluations (Informal Discussions)

Dr Moolman's knowledge of where mathematics is applied in other courses is quite broad. He would always reference a certain profession or another degree of where a certain mathematical principal is applied, but his goal of showing us exactly how it is applied did not only stop in lectures. He would often upload documents that would help us broaden our understanding of the applications which went beyond the scope of the course he lectured us.

Wade van Zyl

Responses from students

His subject has been useful in understanding certain economic and accounting courses which was highly beneficial.

Zoë Duffield

Dr Ruan would often also relate topics to Economics to try to make us understand the wisdom behind the work. This had not only helped with Maths but as well as Economics. Every so often, Dr Ruan would remind us what the working life after university would be like to motivate us to work hard.

Yaseen Osman

Responses from students

Dr Moolman is very good at introducing new topics he firstly connects them with the previous topics and explaining its importance by putting a map of how this topic will impact your studies not only in Mathematics but also in the other subjects. For example, I remember how he introduced partial derivatives and how he connected it so well with the past differentiation and how it will impact our understanding when we get to Mathematical Statistics and Advanced Calculus. I remember how excited I got when I saw partial derivatives in stats it made me more excited about Mathematics and how it is used in other disciplines seeing the impact of Mathematics in the bigger and greater world.

Anele Siyotula

Responses from students

He has also personally stimulated and encouraged me indirectly to continue in Mathematics because he would always refer to higher-level courses in terms of how they unpack the content further but also the application to real life problems almost in every lecture. He makes lectures interesting with respect to how he interlinks what we learn and other fields of study.

Kgaogelo Dlamini

Responses from students

He frequently highlighted the innumerable ways in which mathematical concepts could be applied to almost every real-life context. This not only stimulated intrigue within me, but also made the abstract theories involved in mathematics much more concrete in my mind. As such, I was able to learn with greater ease and proficiency.

Heiletjé van Zyl

Responses from students

While mathematics is not my main focus, it has helped me with discipline and critical thinking in my further studies in Biochemistry and Human anatomy.

Caitlin Stoffels

Dr Moolman shows us how our maths applies in the real world stimulating us to "read the maths" and think for ourselves. He has taken a subject that I once despised and made it exciting and interesting, so much so, that I look forward to attending my mathematics lectures and tutorials.

Sebastian Hermansen

Responses from students

It is helpful that he gives us insight as to where our maths content relates to other courses such as economics.

Throughout high school I had a superficial understanding of maths. However in the past six months my mathematical comprehension has deepened, and I have been able to connect with the subject extensively.

contd…

Responses from students

After spending exactly 27 minutes trying to solve only this one question, the delight after finding the answer surprised me. He has awakened my passion for the subject and instilled an excitement to study, understand and apply the content.

He managed to transform maths from the mundane chore that it once was to the enjoyable challenge that I now know it to be.

Helena Comits

Responses from students

I had previously done a MAM1000W course in my first year where Proof by contradiction and Induction were taught and I never understood it until Dr. Moolman's lectures. He went over implications in detail beforehand, so as to cover the bases of the aforementioned topics. I benefited from those lectures greatly as I found the Proof by Contradiction and Induction chapters much simpler after having such a clear foundation.

contd…

Responses from students

Dr. Moolman also taught the binomial theorem much more depth which benefitted me in my degree as I also learn about this in a Statistics course, his explanation of it gave me further understanding of what I was doing in Statistics.

Diya Ramchuran

Responses from students

Great effort was made to show us that what we were learning, did not exist in a bubble.

He showed us where what we learned would be applied in the real world, e.g. if we were doing an experiment and we obtained a function we did not know the properties of, we could approximate using a sum of polynomials over a certain interval, to better understand the function and thus draw conclusions.

Dr Moolman showed interest in the other courses we took and would often relate the work we did in class to the work we did in our other courses.

Matthew Weppenaar

Responses from students

Doing mathematics can be frustrating at times for any students, and Dr Moolman countered this by continuously expressing the beauty and sophistication of the subject:

he would often include extra resources, separate to the course content, that catered to various scientific branches such as finance, biology, physics and chemistry, to emphasize the usefulness of certain concepts to the variety of different majors attending his lectures,

as well as serve as extra motivation for the studies of mathematics.

William McMichael

Responses from students

At first I was not so certain about my decision but Dr Moolman soon helped me to understand the benefits of this decision. He assured me that it wasn't at all a bad move on my part – and I can now say that he was definitely correct with his optimism towards my academic decisions.

I was studying towards majors unrelated to math explicitly, where now I'm majoring in math.

Zack Pryde

Responses from students

Dr Moolman also had some knowledge on how mathematics can be applied in other sectors like programming with languages such as python, C#, C++, Java and many more. This was very interesting for me because this is part of my area of study, so I was able to program using some mathematical concepts which I found interesting. I was able to understand the uses of concepts used in mathematics and how they can be applied even in our daily life. This made the subject as a whole more sensible and it became enjoyable because I understood it, and this motivated me to do my best in it.

Rufaro Chibanda

Responses from students

Dr Moolman always made use of different mathematical programmes to make the work more understandable for example helping us visualise different graphs by using Desmos and explaining their applications in the financial world. This ensured that students with different learning styles were catered for as we could listen, write and see the outcome in a visual picture. Most learners in Dr Moolman's lectures are commerce students therefore he always made the effort of linking our studies in mathematics to subjects such as economics and the financial industry as a whole and it was quite helpful to receive this information as one could understand the bigger picture.

Koketso Kamogelo Koto

Responses from students

This year I have come to appreciate certain notes you may have uploaded last year as "further reading" especially for my differential equations course. On that same topic I can't give enough thanks to removing a test or two and replacing them with projects, one of which I hope to pursue further in my education - population models and dynamics.

Lastly I would like to express my gratitude for a book you gave on mathematical models. This along with your total encouragement and interest is one of the largest factors as to why I am studying applied maths with such interest. ie) Without you as a lecturer for the last two years I'd probably be trying to convince myself I like chemistry for a third year.

Michael Kievits