INTERPRETING RATE OF CHANGE IN APPLIED CONTEXT:

REFLECTING ON STUDENTS' REASONING

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PRESENTERS

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MSc in Applied Mathematics;

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Tatiana's professional career ranges from teaching and lecturing to publishing and e-publishing. Prior to joining CETAP, she worked at Pearson SA as a Solutions Manager, Learning and Assessment Solutions Specialist and STEM Programme Manager. She was responsible for research, development and project management of various assessment, testing, teacher training and new learning and teaching projects with special focus on Mathematics, Science and e-learning.

Her current interests are in Mathematics assessment, item development and design, as well as the assessment delivery models.

Sanet Steyn

Academic Literacy Research Lead in CETAP, University of Cape Town

MA in Applied Linguistics;

MA in English focus on language assessment and curriculum design

Sanet Steyn's research interests include academic literacy, language assessment and curriculum design. She was involved in both the teaching and assessment of academic literacy since 2012.

In her current role in CETAP she is responsible for the development and quality of tests such as the academic literacy component of National Benchmark Tests. Her research is currently focused on the design and development of parallel instruments for multilingual contexts and the challenges facing test developers working in these spaces.

INTRODUCTION

Objectives:

- Academic literacy in mathematics, in particular calculus applications
- NBT observations in relation to 1st year studies
- Rates of change in the SA CAPS & 1st year curricula
- NSC Report on rates of change questions
- Analysis of questions involving rates of change
- Problem solving strategy using AL

Purpose:

Investigate Academic Literacy demands when using Polya's problem-solving strategy to solve problems with rates of change

STUDENT REASONING FROM AL PERSPECTIVE

- Academic literacy (AL) refers to students' capacity to engage successfully with the demands of academic study (in the medium of instruction) within their educational context.
- To understand student reasoning, we must examine the academic demands of our course content from many different angles and reflect on not only the more obvious, subject-specific skills and knowledge areas that form part of a student's conceptual understanding of a particular component of the subject, but also the abilities that are being used in the background (such as academic literacy skills).
- Research indicates that inadequate preparation in using academic language at school level is an obstacle to learning for students with low language proficiency, especially in relation to conceptual understanding. Careful attention should be given to the specific academic language demands emerging during conceptual development in mathematics learning. [Prediger & Zindel]

ACADEMIC LITERACY SKILLS AT PLAY IN MATHEMATICS

Skill Assessed	Explanation of Skill Area
Cohesion	**Identify links** (anaphoric and cataphoric) and other **mechanisms that connect** parts of text.
Communicative function	**Identify and understand function** of parts of **sentences / discourse**: define; exemplify; support/endorse; contradict; or persuade etc.
Discourse relations	**Understand** the **structure and organisation of discourse and argument**: 1) transitions in argument, logical development; 2) how main idea, supporting ideas / 'evidence' are presented; 3) introductions / conclusions.
Essential / non-essential (Distinction making)	**Make distinctions, classify / categorise and compare:** identify main ideas and supporting detail; cause and effect; statements and examples; facts and opinions; propositions and their arguments; being able to 'label'
Grammar / syntax	**Understand and analyse** the **grammatical and sentence structures** in academic language, and how that **affects / can change meaning**
Extrapolation, application & inferencing	Draw **conclusions** and **apply insights**, either on the basis of what is **stated** in texts or is **implied** by these texts.
Metaphorical expression	**Understand and work with metaphor in language**. This includes the capacity to perceive language connotation, word play, ambiguity, idiomatic expressions, and so on.
Text genre	Can **perceive 'audience'** in text, **purpose** in writing and understand **register** and **tone**: includes an ability to understand text register (formality / informality) and tone (didactic / informative / persuasive / etc.)
Vocabulary	**Derive/work out the meaning of words/terms** from the context

NBT National Benchmark Tests Project

CETAP Centre for Educational Testing for Access and Placement

ACADEMIC LITERACY SKILLS AT PLAY IN MATHEMATICS

Essential / non-essential (Distinction making)	Make distinctions, classify, categorise and compare information provided
Extrapolation, application & inferencing	Make inferences and identify implications, extrapolate facts, apply insights to procedure
Metaphorical expression	Become familiar with typical phrases used in the subject, connotations and jargon
Understanding vocabulary	Understand subject-specific terminology, general academic terms, context descriptors

ACADEMIC LITERACY SKILLS AT PLAY IN MATHEMATICS

Cohesion →

Communicative function →

Discourse relation →

AL Skills related to understanding structure, sequence and function

Similar to understanding of sequence, procedure and function in mathematics

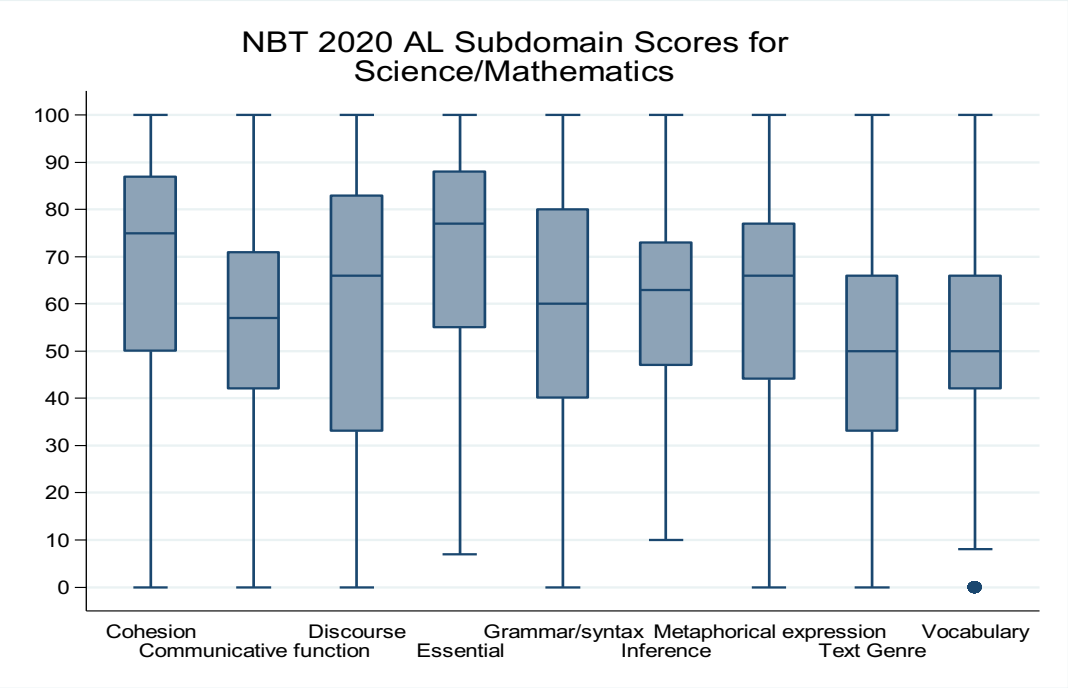
There may be links between these skills, though the specific application of these skills will be limited to tasks requiring written or oral production of texts

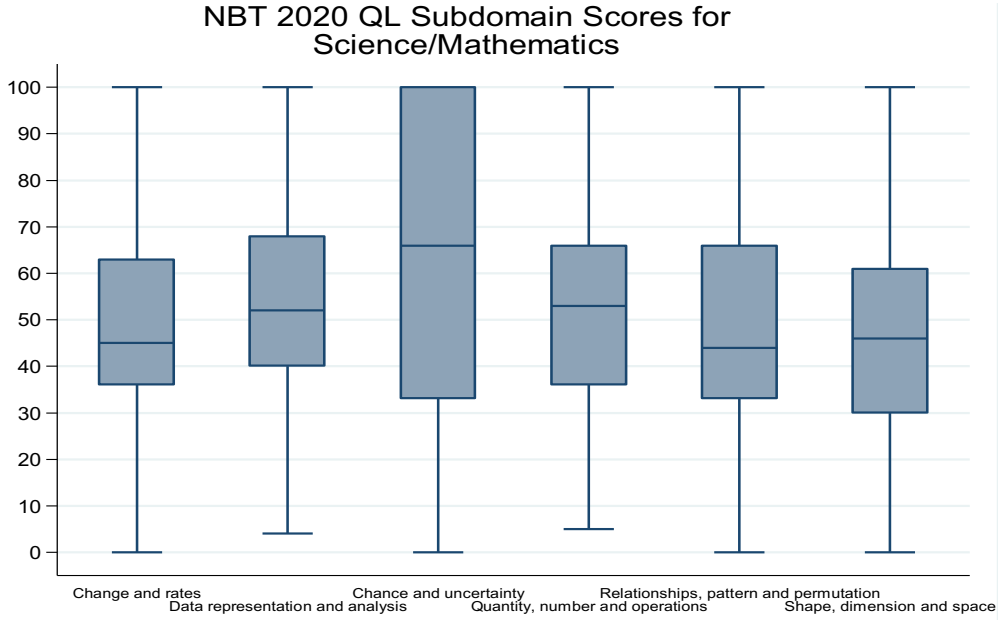
Constructing a basic argument is relevant to all disciplines and perhaps most clearly in mathematics – e.g. knowing what counts as evidence for a claim

ACADEMIC LITERACY SKILLS AT PLAY IN MATHEMATICS

Grammar / syntax	Ability to understand scenarios that rely on noticing grammatical of syntactical elements that influence the meaning of a text may affect the ability to complete a task
Text genre	Understanding of text purpose and audience, appropriate language usage, register and tone Impact mostly limited to tasks requiring written or oral production of texts Ability to understand scenarios that rely on noticing things like an author's bias may affect the ability to complete a task

NATIONAL BENCHMARK TESTING (NBTS): OBSERVATIONS





In general, students seem to struggle with:

1) non-literal language use;
2) terminology and vocabulary items they are unfamiliar with;
3) questions where things are not explicitly stated and require inferencing

In addition to these developmental areas we see from the AL performance, the QL subdomain analysis points specifically to issues in dealing with change and rates in everyday scenarios.

NBT National Benchmark Tests Project

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WHAT WE KNOW FROM THE NSC DIAGNOSTIC REPORT

2017	
Candidates did not realise th about the critical values of th given in the question. Many o about calculating these critica before sketching the graph. S joined the points with straight curves.	s confused 'how many times' e' in answering Q8.2. They as t = 6 instead of indicating ached the floor only once.
Expose learners to higher-ord questions and interpretation o Initially teachers should assist understanding what is being looks like on the picture and are relevant to the interval re solution.	o expose learners to word er for them to gain confidence.
…	

Summary of observations

1) Students seem to be missing signals in questions.
2) Understanding and interpreting the questions are highlighted as problem areas.
3) Issues with understanding the language used in questions are specifically mentioned.

In addition to the recommendation that teachers expose learners to more higher-order thinking questions, we need to find a way of anticipating where learners would need additional guidance AND what that guidance should entail.

UNPACKING THE ACADEMIC DEMANDS OF SUBJECT CONTENT, MATERIALS, ASSESSMENTS

Course content, teaching and learning materials	Assessments, tasks and exercises
1 - Reading and analysing the content • Identify key vocabulary items (action words; terminology; context descriptors) • Determine what the focus of the content is.	1 - Reading and analysing the task • Identify key vocabulary items (action words; terminology; context descriptors) • Determine what the exercise is asking you to do? • Extrapolate information from what is presented and make initial inferences.
2 - Inferencing, distinction-making, linking to content knowledge • Determine what is new and what is based on familiar concepts. • How do they differ and how are they the same? • Analyse examples like you would an assessment, task or exercise.	2 - Inferencing, distinction-making, linking the exercise to content knowledge • What do you know? (Identify the facts.) • Which mathematical principles/concepts are involved? • Sift through everything to focus only on what you need to complete the task. (Essential vs. Non-essential; Relevant vs. Irrelevant etc.)
3 - Identify standard procedure for applying this knowledge. • Take note of the format of typical answers to exercises that relate to this content.	3 - Complete the task • Apply mathematical knowledge and follow standard procedure. • Take note of the format of the answer that the exercise requires.

CAPS & 1st YEAR: RATES OF CHANGE

CAPS Functions:

- domain and range, intercepts with the axes,
- turning points, minima, maxima,
- asymptotes (horizontal and vertical), shape and symmetry,
- average gradient (average rate of change),
- intervals on which the function increases /decreases.

CAPS Calculus:

- An intuitive understanding of the limit concept, in the context of approximating the rate of change or gradient of a function at a point.
- Solve practical problems concerning optimisation and rate of change, including calculus of motion.

1st year Mathematics: Derivatives and Rates of Change

1st year Physics: Kinematics in one dimension; position, time and displacement, velocity, linear acceleration, motion with constant acceleration, free-fall and motion on an inclined plane.

RATES OF CHANGE PROBLEMS THROUGH AL LENS

Calculus is an advanced mathematics topic and the applications of rates of change are used in many courses, not only mathematics:

- Rates of change with respect to time:
  - Rate of change of work with respect to time (*power* in physics)
  - Rate of change in concentration of a reactant with respect to time (*rate of reaction* in chemistry)
  - Rate of change of displacement with respect to time (*velocity* in physics)
  - Rate of change of bacteria population with respect to time (*population growth* in biology)
- Rates of change of quantities:
  - Rate of change of production cost with respect to the number of items produced (economics)

Academic Literacy & concept development

Change

$$f(a+h) - f(a)$$

Perceptible differences from one case to another.

→ Rate of change

There is a pattern to these differences that can be described as a rate.

→ Average rate of change

$$\frac{f(a+h) - f(a)}{h}.$$

→ Instantaneous rate of change

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

There are different ways of looking at the pattern (rate) in these differences.

PROBLEM SOLVING STRATEGY USING AL

Four steps approach:

Step 1: Construct learners' understanding of the problem (extrapolating information/inferencing; developing reasoning)

Step 2: Link to mathematical content knowledge, use of concepts (draw inferences & select relevant information)

Step 3: Complete the task; application of knowledge & presentation of the argument

Step 4: Reflect with specific focus on potential problems in student's reasoning

George Polya's four steps:
1 – Understand the problem
2 – Think of a plan
3 – Carry out the plan
4 – Look back

SOLUTION WITH TEACHING AND LEARNING STEPS

Step 1: Construct learners' understanding of the problem (extrapolating information/inferencing; developing reasoning)

114 CHAPTER 2 Derivatives

14. If a rock is thrown upward on the planet Mars with a velocity of 10 m/s, its height (in meters) after t seconds is given by $H = 10t - 1.86t^2$.

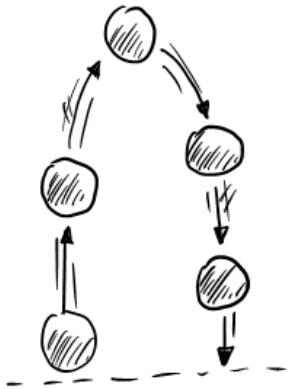
(a) Find the velocity of the rock after one second.

(b) Find the velocity of the rock when $t = a$.

(c) When will the rock hit the surface? ⇨ If H = 0, solve for t.

(d) With what velocity will the rock hit the surface?

⇨ Find the velocity of the rock when H = 0. [This is based on (c) previous answer]



(c) and (d) use language related to the scenario rather than using standard prompts. The student, therefore, must make the necessary inferences to unpack the question and interpret the task at hand.

From the problem:

Rock is thrown

How heavy is the rock? ⇨ Not provided

Planet Mars

Velocity of the rock is 10 m/s

Height is measured in meters

Height of the rock is H ⇨ Use of notation

Equation for H is given

From the questions:

The rock has a velocity

The rock is in the air & has a height

The height is changing

The rock hits the surface

The time is changing

(a) Velocity (v) after one second:

1 second from the time the rock is thrown

(b) Velocity (v) when $t = a$:

The time is some arbitrary a

(c) "When" means looking for the time, when the rock hits the surface:

The height is 0

(d) Velocity when the rock hits the surface:

The velocity when the height = 0

Step 2: Link to mathematical content knowledge, use of concepts (draw inferences & select relevant information)

(a) Find the velocity of the rock after one second. Given the height equation, the velocity is the 1st derivative "After 1 second" means time $t = 1$ sec	$H = 10t - 1{,}86t^2$ $v = H'$ $t = 1\ s$
(b) Find the velocity of the rock when $t = a$ Found velocity in (a)	$v = H'$ $t = a$
(c) When will the rock hit the surface? 'Hit the surface', so $H = 0$ Substitute $H = 0$ and solve for t	$H = 10t - 1{,}86t^2$ $H = 0\ m$ $0 = 10t - 1{,}86t^2$
(d) With what velocity will the rock hit the surface? Already have velocity from (a) Found time in (c)	$v = H'$ $t\ s$

From the problem:

Rock is thrown

~~How heavy is the rock?~~

~~Planet Mars~~

~~Velocity of the rock is 10 m/s~~

Height is measured in meters

Height of the rock is H

Equation for H is given

Velocity

Height

Displacement

Time

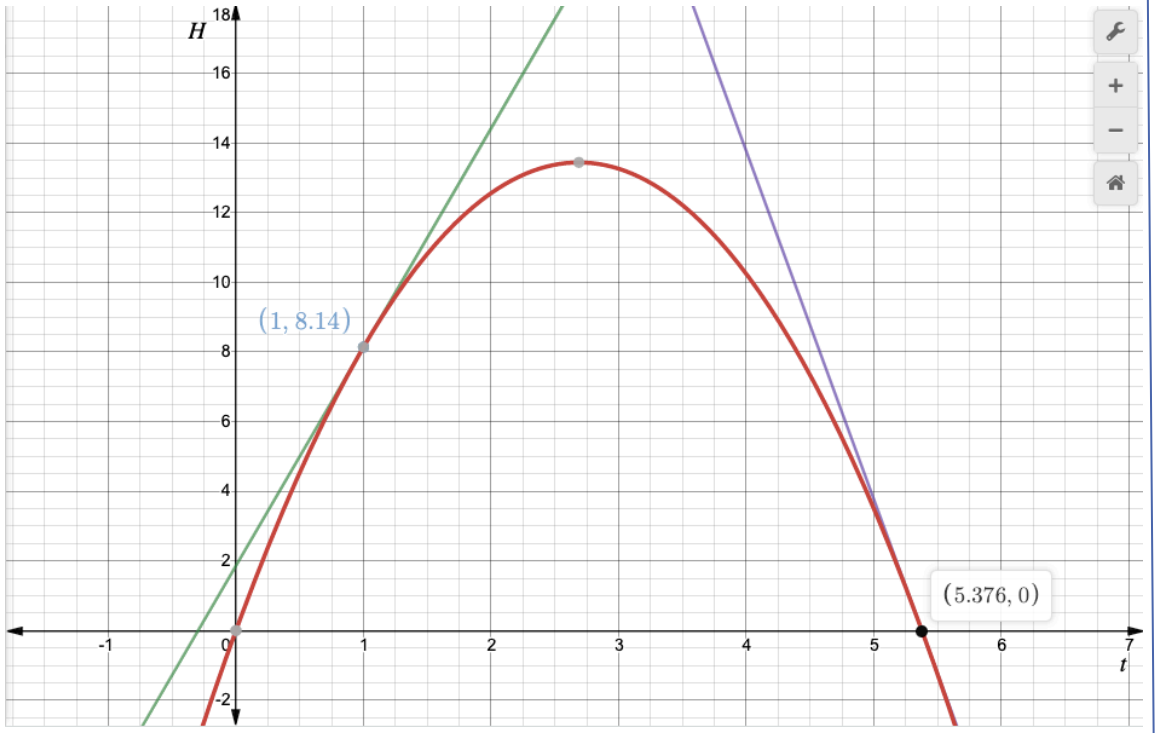
Differentiation

Step 3: Complete the task; application of knowledge & presentation of the argument

(a) Find the velocity of the rock after one second. Find the 1st derivative of H Substitute $t = 1$ sec	$H = 10t - 1{,}86t^2$ $v = H' = 10 - 3{,}72t$ $t = 1\ s$ $v = 10 - 3{,}72(1) = 6{,}28\ m/s$	Ability to differentiate Ability to substitute a numeric value of the variable Ability to calculate with decimal numbers/Ability to use a calculator
(b) Find the velocity of the rock when $t = a$ Use $t = a$ to substitute into v	$v = H' = 10 - 3{,}72t$ $t = a$ $v = 10 - 3{,}72a$	Ability to substitute the numeric value of a variable
(c) When will the rock hit the surface? Substitute $H = 0$ and solve for t Interpret the result	$H = 10t - 1{,}86t^2$ $0 = 10t - 1{,}86t^2$ $0 = t(10 - 1{,}86t)$ $t = 0\ s$ or $t = \frac{10}{1{,}86} = 5{,}38\ s$ The rock will hit the surface after 5,38 seconds.	Ability to solve quadratic equation (without a free term) Ability to factorise a binomial Ability to interpret the answers and choose the correct solution Ability to divide by a decimal number/Ability to use a calculator
(d) With what velocity will the rock hit the surface? Found time in (c) Use this time to substitute into v	$v = H' = 10 - 3{,}72t$ $t = 5{,}38\ s$ $v = 10 - 3{,}72 \times 5{,}38 = -10{,}01\ m/s$	Ability to substitute Ability to multiply and subtract decimal numbers/Ability to use a calculator

Step 4: Teacher reflection with specific focus on potential problems in learner's reasoning

Which AL skills need to be in place?
Did students understand the context?
What (other) Mathematical models could be designed by this task?
Did students understand the mathematical meaning of the task?
What Maths skills students are missing?
Are there any misconceptions?
Link to further concepts?



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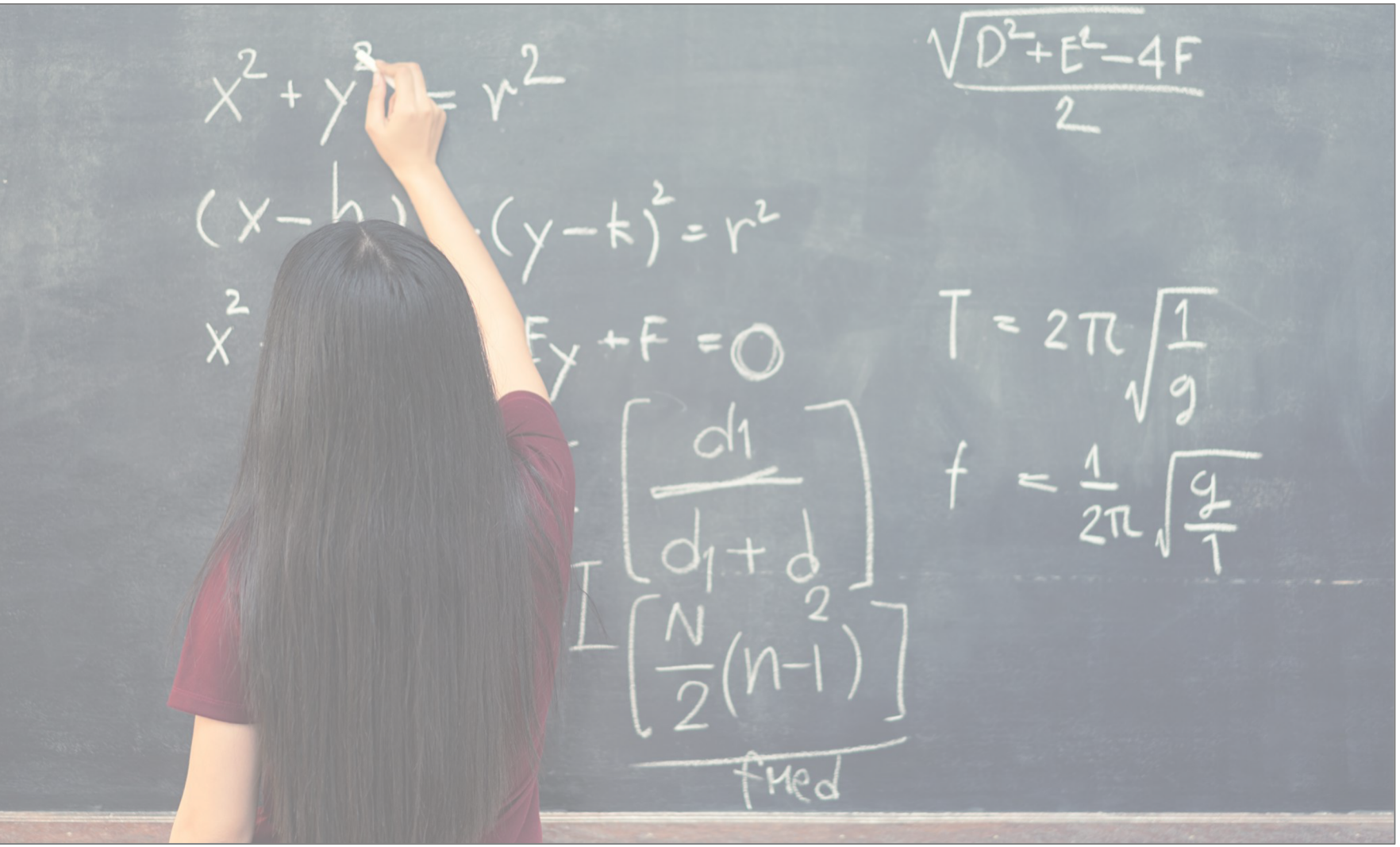
(a) Find the velocity of the rock after one second.
(b) Find the velocity of the rock when $t = a$.
(c) When will the rock hit the surface?
(d) With what velocity will the rock hit the surface?

(a) Found height instead of velocity after 1 second; gave initial velocity as the answer;
(b) Found height instead of velocity when $t = a$; trying to solve for a; giving numerical values for a;
(c) Substituted t = 0 into the height equation; used H = 0, but could not solve the equation for t
(d) Used v = 10 m/s as the answer; substituted 10 m/s into the height equation;



CONCLUSIONS

UNDER INVESTIGATION….

What can diagnostic information tell us about AL's preparedness for teaching/using problem solving strategies in Mathematics?

How can AL subdomains be integrated in the teaching of problem-solving in mathematics?

How do AL subdomains/skills develop students' conceptual understanding?

Which AL subdomains play the most important role in applications of Polya's stages of problem-solving?

REFERENCES & RESOURCES

- Past Exam papers (Grade 8 – 12, incl. provincial) + Memos: https://edwardsmaths.com/
- NSC Reports: https://dbedashboard.co.za/School-Improvement/problem-solving/nsc-subject-diagnostic-reports/
- NBT Reports (Institutions, Reports): https://www.nbt.ac.za/
- Stuart, James. Calculus, Metric version 8E. USA, Cengage Learning, 2016
- Susanne Prediger, Carina Zindel, TU Dortmund University, Germany, 2017. EURASIA Journal of Mathematics Science and Technology Education, 13(7b):4157-4188. *School Academic Language Demands for Understanding Functional Relationships: A Design Research Project on the Role of Language in Reading and Learning*

THANK YOU

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